MT001: Taking the Mystery out of the Infamous Formula, "SNR=6.02N + 1.76dB," and Why You Should Care

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1 author:

Walt Kester
Analog Devices, Inc.

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by Walt Kester

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INTRODUCTION

You don't have to deal with ADCs or DACs for long before running across this often quoted formula for the theoretical signal-to-noise ratio (SNR) of a converter. Rather than blindly accepting it on face value, a fundamental knowledge of its origin is important, because the formula encompasses some subtleties which if not understood can lead to significant misinterpretation of both data sheet specifications and converter performance.

Remember that this formula represents the theoretical performance of a perfect N-bit ADC. You can compare the actual ADC SNR with the theoretical SNR and get an idea of how the ADC stacks up.

This tutorial first derives the theoretical quantization noise of an N-bit analog-to-digital converter (ADC). Once the rms quantization noise voltage is known, the theoretical signal-to-noise ratio (SNR) is computed. The effects of oversampling on the SNR are also analyzed.

DERIVATION

The maximum error an ideal converter makes when digitizing a signal is ±½ LSB as shown in the transfer function of an ideal N-bit ADC (Figure 1). The quantization error for any ac signal which spans more than a few LSBs can be approximated by an uncorrelated sawtooth waveform having a peak-to-peak amplitude of q, the weight of an LSB. Another way to view this approximation is that the actual quantization error is equally probable to occur at any point within the range ±½ q. Although this analysis is not precise, it is accurate enough for most applications.

W. R. Bennett of Bell Laboratories analyzed the actual spectrum of quantization noise in his classic 1948 paper (Reference 1). With the simplifying assumptions previously mentioned, his detailed mathematical analysis simplifies to that of Figure 1. Other significant papers and books on converter noise followed Bennett's classic publication (References 2-6).

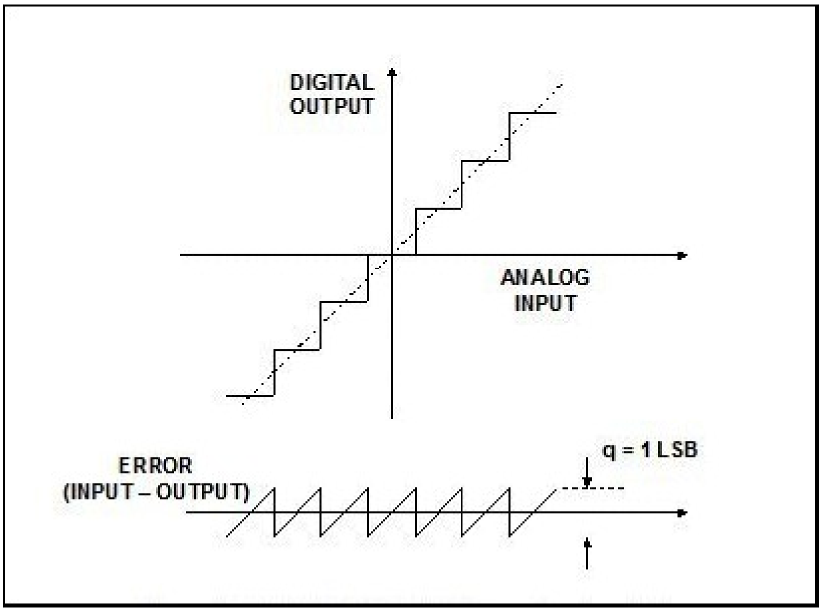

Figure 1: Ideal N-bit ADC Quantization Noise

The quantization error as a function of time is shown in more detail in Figure 2. Again, a simple sawtooth waveform provides a sufficiently accurate model for analysis. The equation of the sawtooth error is given by

$$e(t) = st,\ -q/2s < t < +q/2s. \qquad \text{Eq. 1}$$

The mean-square value of e(t) can be written:

$$\overline{e^2(t)} = \frac{s}{q}\int_{-q/2s}^{+q/2s}(st)^2\,dt\,. \qquad \text{Eq. 2}$$

Performing the simple integration and simplifying,

$$\overline{e^2(t)} = \frac{q^2}{12}\,. \qquad \text{Eq. 3}$$

The root-mean-square quantization error is therefore

$$\text{rms quantization noise} = \sqrt{\overline{e^2(t)}} = \frac{q}{\sqrt{12}}\,. \qquad \text{Eq. 4}$$

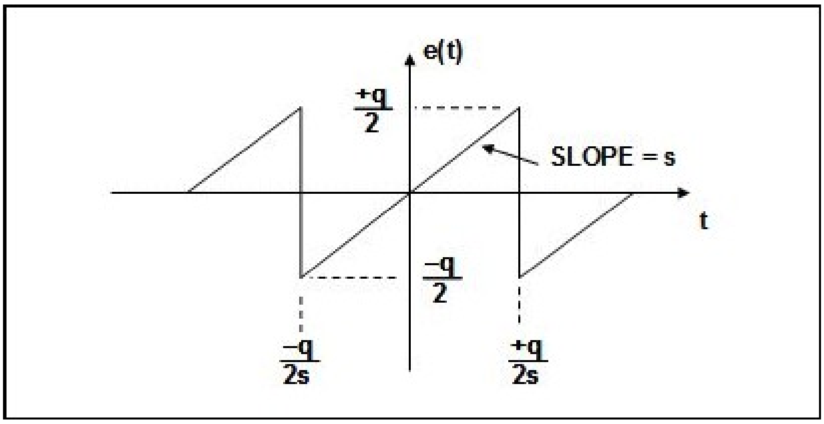

Figure 2: Quantization Noise as a Function of Time

The sawtooth error waveform produces harmonics which extend well past the Nyquist bandwidth of dc to $f_s/2$. However, all these higher order harmonics must fold (alias) back into the Nyquist bandwidth and sum together to produce an rms noise equal to $q/\sqrt{12}$.

As Bennett points out (Reference 1), the quantization noise is approximately Gaussian and spread more or less uniformly over the Nyquist bandwidth dc to $f_s/2$. The underlying assumption here is that the quantization noise is uncorrelated to the input signal. Under certain conditions where the sampling clock and the signal are harmonically related, the quantization noise becomes correlated, and the energy is concentrated in the harmonics of the signal − however, the rms value remains approximately $q/\sqrt{12}$. The theoretical signal-to-noise ratio can now be calculated assuming a full-scale input sinewave:

$$\text{Input FS Sinewave} = v(t) = \frac{q2^N}{2}\sin(2\pi ft). \qquad \text{Eq. 5}$$

The rms signal of the input signal is therefore

$$\text{rms value of FS input} = \frac{q2^N}{2\sqrt{2}}\,. \qquad \text{Eq. 6}$$

The rms signal-to-noise ratio for an ideal N-bit converter is therefore

$$SNR = 20\log_{10}\frac{\text{rms value of FS input}}{\text{rms value of quantization noise}} \qquad \text{Eq. 7}$$

$$SNR = 20\log_{10}\left[\frac{q2^N/2\sqrt{2}}{q/\sqrt{12}}\right] = 20\log_{10}2^N + 20\log_{10}\sqrt{\frac{3}{2}} \qquad \text{Eq. 8}$$

$$SNR = 6.02N + 1.76\text{dB}, \quad \text{over the dc to } f_s/2 \text{ bandwidth.} \qquad \text{Eq. 9}$$

Bennett's paper shows that although the actual spectrum of the quantization noise is quite complex to analyze, the simplified analysis which leads to Eq. 9 is accurate enough for most purposes. However, it is important to emphasize again that the rms quantization noise is measured over the full Nyquist bandwidth, dc to $f_s/2$.

OVERSAMPLING AND UNDERSAMPLING

In many applications, the actual signal of interest occupies a smaller bandwidth, BW, which is less than the Nyquist bandwidth (see Figure 3). If digital filtering is used to filter out noise components outside the bandwidth BW, then a correction factor (called process gain) must be included in the equation to account for the resulting increase in SNR as shown in Eq. 10.

$$SNR = 6.02N + 1.76\text{ dB} + 10\log_{10}\frac{f_s}{2\cdot BW}, \quad \text{over the bandwidth BW.} \qquad \text{Eq. 10}$$

The process of sampling a signal at a rate which is greater than twice its bandwidth is referred to as oversampling. Oversampling in conjunction with quantization noise shaping and digital filtering are the key concepts in sigma-delta converters, although oversampling can be used with any ADC architecture.

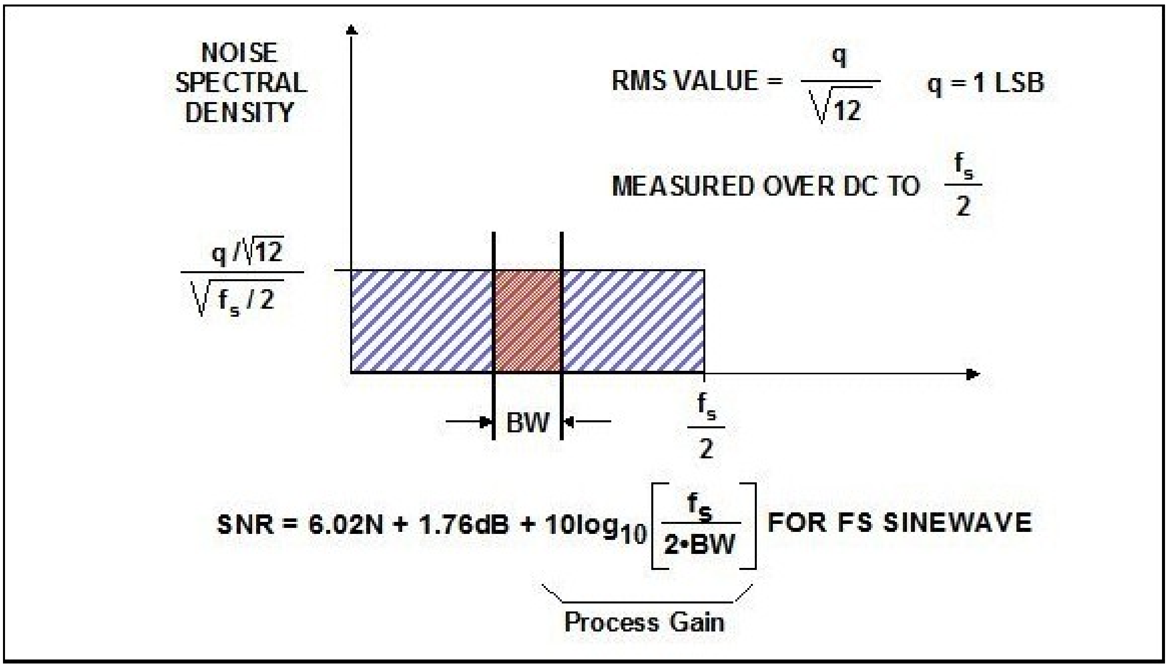

Figure 3: Quantization Noise Spectrum Showing Process Gain

The significance of process gain can be seen from the following example. In many digital basestations or other wideband receivers the signal bandwidth is composed of many individual channels, and a single ADC is used to digitize the entire bandwidth. For instance, the analog cellular radio system (AMPS) in the U.S. consists of 416 30-kHz wide channels, occupying a bandwidth of approximately 12.5 MHz. Assume a 65-MSPS sampling frequency, and that digital filtering is used to separate the individual 30-kHz channels. The process gain due to oversampling for these conditions is given by: